

The $a_1(1260)$ meson and chiral symmetry restoration and deconfinement at finite temperature QCD

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Abstract

We consider the light-quark axial-vector current correlator in the framework of thermal QCD sum rules to: (a) find a relation between chiral-symmetry restoration and deconfinement, and (b) determine the temperature behaviour of the $a_1(1260)$ width and coupling. Our results show that deconfinement takes place at a slightly lower temperature than chiral-symmetry restoration. This difference is not significant given the accuracy of the method. The behaviour of the $a_1(1260)$ parameters is consistent with quark-gluon deconfinement, since the width grows and the coupling decreases with increasing temperature.

Keywords: Finite temperature field theory, hadron physics.

1. Introduction

A quark-gluon deconfinement phenomenological order parameter was first introduced in the vector channel, and in the framework of thermal QCD sum rules in [1]. It is given by the squared energy threshold, $s_0(T)$, for the onset of perturbative QCD (PQCD) in the hadronic spectral function. In general, around $s_0(T = T_c)$, the resonances in the spectrum, in any channel, will be either no longer present or will become very broad. When the temperature approaches the critical value for deconfinement, T_c , one would expect hadrons to disappear from the spectral function, leading to a description based entirely on PQCD. It was shown in [1] that $s_0(T)$ in the vector channel is a decreasing function of the temperature, together with the coupling of the ρ -meson to the vector current. This analysis, however, was performed only in the zero-width approximation. Resonance broadening was proposed in [2]-[3], and subsequently confirmed in other applications [4], as an important phenomenological information for the deconfinement transition.

QCD sum rules in the axial-vector channel, [2], established a link between deconfinement and chiral-symmetry restoration at finite temperature. This discussion was improved later in [5], and recently updated and extended also to finite density in [6]. These analyses indicate that the temperature at which $s_0(T)$ vanishes is very close to that at which the quark condensate vanishes. The results suggest that both phase transitions take place at roughly the same temperature. The analyses of [2], [5]-[6] made use of the finite energy QCD sum rule (FESR) of the lowest dimension ($d = 2$) in the axial-vector channel, assuming full saturation of the hadronic spectral function by the pion pole. This assumption is not justified if one were to consider the next two FESR of dimension $d = 4$ and $d = 6$. In fact, already at $T = 0$ one finds that the values of the condensates of dimension $d = 4$ and $d = 6$ that follow from the second and third FESR are barely consistent with results obtained from experimental data [7]. This suggests the presence of additional hadronic contributions. In fact, the data in this channel include also the $a_1(1260)$ resonance.

Using the first three FESR, we will reconsider the light-quark axial-vector channel using a hadronic spectral function involving the pion pole as well as the $a_1(1260)$ resonance. We will obtain information on the temperature behavior of the $a_1(1260)$ coupling and hadronic width. The results show that $s_0(T)$ vanishes at a critical temperature some 10% below that for chiral-symmetry

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restoration. This difference is not relevant within the accuracy of the method. The $a_1(1260)$ coupling initially increases with increasing T up to $T/T_c \approx 0.7$, and then decreases sharply up to T_c . Finally, the hadronic width of the $a_1(1260)$ remains constant up to $T/T_c \approx 0.6$, increasing sharply thereafter. These behaviours are consistent with a quark-gluon deconfinement scenario.

2. FESR at $T = 0$

We consider the correlator of light-quark axial-vector currents

$$\begin{aligned} \Pi_{\mu\nu}(q^2) &= i \int d^4x e^{iqx} \langle 0 | T(A_\mu(x), A_\nu^\dagger(0)) | 0 \rangle \\ &= -g_{\mu\nu} \Pi_1(q^2) + q_\mu q_\nu \Pi_0(q^2), \end{aligned} \quad (1)$$

where $A_\mu(x) =: \bar{d}(x) \gamma_\mu \gamma_5 u(x) :$ is the (charged) axial-vector current, and $q_\mu = (\omega, \vec{q})$ is the four-momentum carried by the current. Concentrating on e.g. $\Pi_0(q^2)$ and invoking the Operator Product Expansion (OPE) of current correlators at short distances, one has

$$4\pi^2 \Pi_0(q^2)|_{\text{QCD}} = C_0 \hat{I} + \sum_{N=1} \frac{C_{2N}(q^2, \mu^2)}{Q^{2N}} \langle \hat{O}_{2N}(\mu^2) \rangle, \quad (2)$$

where $Q^2 \equiv -q^2$, $\langle \hat{O}_{2N}(\mu^2) \rangle \equiv \langle 0 | \hat{O}_{2N}(\mu^2) | 0 \rangle$, μ^2 is a renormalization scale, the Wilson coefficients C_N depend on the Lorentz indexes and quantum numbers of the currents, and on the local gauge invariant operators \hat{O}_N built from the quark and gluon fields. The Wilson coefficients are calculable in PQCD. The unit operator above has dimension $d = 0$ and $C_0 \hat{I}$ stands for the purely perturbative contribution. The dimension $d = 4$ term, a renormalization group invariant quantity, is given by

$$C_4 \langle \hat{O}_4 \rangle = \frac{\pi}{6} \langle \alpha_s G^2 \rangle + 2\pi^2 (m_u + m_d) \langle \bar{q}q \rangle, \quad (3)$$

where the second term is negligible in comparison with the gluon condensate. The leading power correction of dimension $d = 6$ is the four-quark condensate. In the vacuum saturation approximation it becomes

$$C_6 \langle \hat{O}_6 \rangle = \frac{704}{81} \pi^3 \alpha_s |\langle \bar{q}q \rangle|^2, \quad (4)$$

having a mild dependence on the renormalization scale. This approximation has no theoretical justification. Hence, there is no reliable way of estimating corrections, which appear to be rather large from comparisons between Eq. (4) and determinations from data [7]. This poses no problem for the finite temperature analysis,

where Eq.(4) is only used to normalize results at $T = 0$, and one is interested in the behavior of ratios. Cauchy's theorem in the complex squared energy s -plane, leads us to the FESR (at leading order in PQCD)

$$\begin{aligned} (-)^{(N-1)} C_{2N} \langle \hat{O}_{2N} \rangle &= 4\pi^2 \int_0^{s_0} ds s^{N-1} \frac{1}{\pi} \text{Im} \Pi_0(s)|_{\text{HAD}} \\ &- \frac{s_0^N}{N} [1 + O(\alpha_s)] \quad (N = 1, 2, \dots). \end{aligned} \quad (5)$$

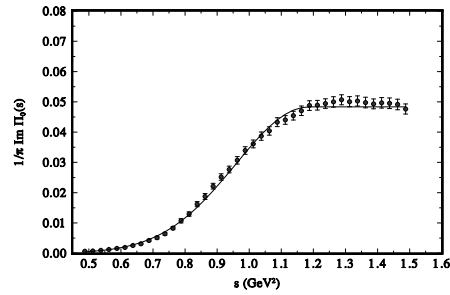


Figure 1: The ALEPH data in the axial-vector channel [9], and in the resonance region together with our fit in the region relevant to the FESR.

The normalization of the correlator in PQCD is

$$\text{Im} \Pi_0(s)|_{\text{QCD}} = \frac{1}{4\pi} [1 + O(\alpha_s(s))]. \quad (6)$$

At $T = 0$ the radiative corrections above are known up to five-loop order, i.e. $O(\alpha_s^4)$, in PQCD. Higher dimensional condensates are poorly known [7] and will not be considered here.

In the hadronic sector the spectral function involves the pion pole followed by the $a_1(1260)$ resonance

$$\text{Im} \Pi_0(s)|_{\text{HAD}} = 2\pi f_\pi^2 \delta(s) + \text{Im} \Pi_0(s)|_{a_1}, \quad (7)$$

where $f_\pi = 92.21 \pm 0.14$ MeV [8] is the pion decay constant and the pion mass has been neglected. We have made a fit to the ALEPH data [9] (at $T=0$); for details, see [10]. This fit together with the ALEPH data is shown in Fig.1 up to $s \approx 1.5 \text{ GeV}^2$ (the FESR determine $s_0 = 1.44 \text{ GeV}^2$). The pion decay constant is related to the quark condensate through the Gell-Mann-Oakes-Renner relation

$$2f_\pi^2 M_\pi^2 = -(m_u + m_d) \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle. \quad (8)$$

Chiral corrections to this relation are at the 5% level [11]. At finite temperature deviations are negligible except very close to the critical temperature [12].

The first three FESR can now be used to determine the PQCD threshold s_0 , and the $d = 4$ and $d = 6$ condensates. These results will be used to normalize all finite temperature results. The value of s_0 obtained by saturating the hadronic spectral function with only the pion pole, and to leading order in PQCD, is $s_0 \simeq 0.7 \text{ GeV}^2$, as in [2], [5]-[6]. This value increases to $s_0 = 1.15 \text{ GeV}^2$ once the $a_1(1260)$ contribution is taken into account, and becomes $s_0 = 1.44 \text{ GeV}^2$ with PQCD to five-loop order.

3. Finite Energy QCD Sum Rules at $T \neq 0$

The extension of the QCD sum rule method to finite

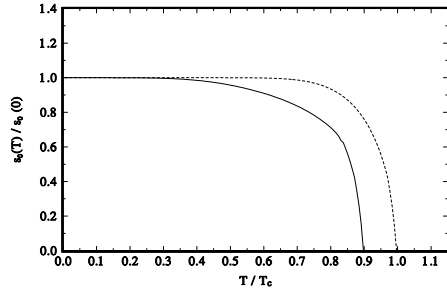


Figure 2: The continuum threshold $s_0(T)/s_0(0)$ signaling deconfinement (solid curve) as a function of T/T_c , together with $f_\pi^2(T)/f_\pi^2(0) = \langle \bar{q}q \rangle(T)/\langle \bar{q}q \rangle(0)$ signaling chiral-symmetry restoration (dotted curve).

temperature implies a T -dependence in the OPE, as well as in the hadronic parameters. The Wilson coefficients and the vacuum condensates become temperature dependent. The strong coupling $\alpha_s(Q^2, T)$ now depends on two scales, the ordinary QCD scale Λ_{QCD} associated with the momentum transfer, and the critical temperature scale T_c associated with temperature. This poses no problems in the asymptotic freedom region and at very high temperatures, $T \gg T_c$, where PQCD can be applied. However, the QCD sum rule method approaches T_c from below, so that the presence of this second scale is problematic. No satisfactory solution to this problem exists, so that analyses must be carried out at leading order in PQCD. At this order in PQCD there are two thermal corrections to Eq.(6). In the time-like region ($q^2 > 0$), we have the annihilation term which in the static limit ($\mathbf{q} \rightarrow 0$) is

$$\text{Im } \Pi_0^+(\omega, T) = \frac{1}{4\pi} \left[1 - 2 n_F\left(\frac{\omega}{2T}\right) \right]. \quad (9)$$

In the space-like region we have the scattering term, which is associated to a cut centered at the origin on the real axis in the complex energy $\omega \equiv \sqrt{s}$ -plane of width $-|\mathbf{q}| \leq \omega \leq |\mathbf{q}|$ [1]. In the static limit this is given by

$$\begin{aligned} \text{Im } \Pi_0^-(\omega, T) &= \frac{4}{\pi} \delta(\omega^2) \int_0^\infty y n_F\left(\frac{y}{T}\right) dy \\ &= \frac{\pi}{3} T^2 \delta(\omega^2), \end{aligned} \quad (10)$$

where $n_F(z) = 1/(1 + e^{-z})$ is the Fermi thermal function, and the chiral limit was assumed. These perturbative results are valid for $T \geq 0$ at the one-loop level in QCD, so that temperature effects develop smoothly from their $T = 0$ values. Non-perturbative contributions will be added later in the framework of the OPE.

In the hadronic sector and at finite temperature, masses, couplings, and widths become T -dependent. Hadronically stable particles, e.g. the pion, with $\Gamma(0) = 0$ develop a width. The important parameters signaling deconfinement are the hadronic width and coupling, but not the mass. A vanishing mass at $T = T_c$ would not signal deconfinement, unless the width diverges at such a temperature. But then the value of the mass becomes irrelevant. This is actually what QCD sum rule analyses show [4]. A notable exception are the scalar, pseudoscalar, and vector charm-anti-charm states which survive beyond T_c [13].

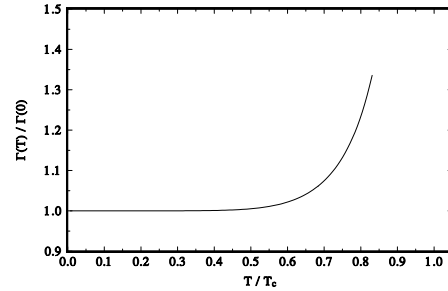


Figure 3: The hadronic width of the $a_1(1260)$ resonance $\Gamma_{a_1}(T)/\Gamma_{a_1}(0)$ as a function of T/T_c .

The temperature behavior of the quark condensate, equivalently f_π^2 , was obtained, in the chiral limit, in the framework of the Schwinger-Dyson equation [14], and for finite quark masses from a fit to lattice QCD results [6], [15]. See [10] for details. The critical temperature is $T_c = 197 \text{ MeV}$. In this temperature region the quark condensate in the chiral limit is essentially the same as

that for finite quark masses. The gluon condensate was obtained from a fit to lattice QCD determinations [16], adjusted to $T_c = 197$ MeV. The first three thermal FESR become

$$8\pi^2 f_\pi^2(T) = \frac{4}{3}\pi^2 T^2 + \int_0^{s_0(T)} ds \left[1 - 2n_F\left(\frac{\sqrt{s}}{2T}\right) \right] - 4\pi^2 \int_0^{s_0(T)} ds \frac{1}{\pi} \text{Im} \Pi_0(s, T)|_{a_1}, \quad (11)$$

$$-C_4 \langle \hat{O}_4 \rangle(T) = 4\pi^2 \int_0^{s_0(T)} ds s \frac{1}{\pi} \text{Im} \Pi_0(s)|_{a_1} - \int_0^{s_0(T)} ds s \left[1 - 2n_F\left(\frac{\sqrt{s}}{2T}\right) \right] \quad (12)$$

$$C_6 \langle \hat{O}_6 \rangle(T) = 4\pi^2 \int_0^{s_0(T)} ds s^2 \frac{1}{\pi} \text{Im} \Pi_0(s)|_{a_1} - \int_0^{s_0(T)} ds s^2 \left[1 - 2n_F\left(\frac{\sqrt{s}}{2T}\right) \right]. \quad (13)$$

These equations determine the continuum threshold $s_0(T)$, the coupling of the $a_1(1260)$ to the axial-vector current, $f_{a_1}(T)$, and its width $\Gamma_{a_1}(T)$, using as input the thermal quark condensate (or $f_\pi^2(T)$), the thermal $d = 4, 6$ condensates, and assuming the $a_1(1260)$ mass to be temperature independent, as supported by results in many channels [4], [13], [17].

4. Results and Conclusions

The FESR have solutions for the three parameters, $s_0(T)$, $f_{a_1}(T)$, and $\Gamma_{a_1}(T)$, up to $T \simeq (0.85 - 0.90) T_c$, a temperature at which $s_0(T)$ reaches its minimum. An inspection of Eq.(11) shows that disregarding the $a_1(1260)$ contribution, $s_0(T)$ would vanish at a lower critical temperature than $f_\pi(T)$ (or $\langle \bar{q}q \rangle(T)$). Making the rough approximation of neglecting the thermal factor $n_F(\sqrt{s}/2T)$ in the second term on the r.h.s. of Eq.(11) leads to $s_0(T) \simeq 8\pi^2 f_\pi^2(T) - (4/3)\pi^2 T^2$. This feature remains valid even after including the $a_1(1260)$ in the FESR, as shown in Fig.2, corresponding to the solution for $s_0(T)$ using all three FESR. This 10% difference is well within the accuracy of the method. The behavior of the width is shown in Fig.3, and that of the coupling in Fig.4. The rise of the width, and the fall of the coupling are indicative of a transition to a quark-gluon deconfined phase at $T = T_c$.

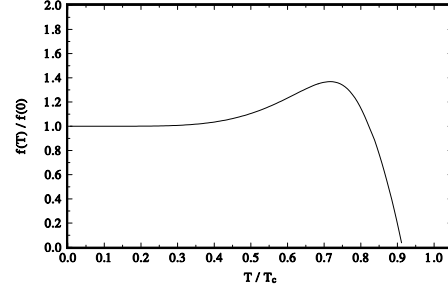


Figure 4: The coupling of the $a_1(1260)$ resonance $f_{a_1}(T)/f_{a_1}(0)$ as a function of T/T_c .

5. Acknowledgments

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